THE TEMPERATURE FIELD IN A BODY BOUNDED BY CONICAL SURFACES DUE TO AN INSTANTANEOUS ANNULAR HEAT SOURCE

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The authors investigate the influence of the internal cavity and apex angle of a cone on the temperature field due to an instantaneous annular source.

In many cases the analytical solution of problems of heat propagation in welding and smelting, mechanical working, heat treatment, etc., is based on the theory of fast-moving high-power sources [1, 2]. The temperature field is determined by summation of the elementary temperature fields due to instantaneous heat sources appropriately distributed. For example, in the arc welding of circular cylinders along a ring or spiral of small pitch, the heating process may be represented as the sum of the heating processes due to annular concentrated sources of an intensity q equal to the product of the effective source power and the time for one revolution [3].

For bodies bounded by conical surfaces (including a hollow cylinder, which is a special case of a conical body), the analytical solution is very laborious, which restricts the application of the theory [1, 2].

It may be assumed that, for bodies which are slightly conical, at sufficiently large distances from the apex the propagation of heat from an instantaneous source, even some time after the application of the pulse, will be approximately the same as that for a circular cylinder of corresponding dimensions. For large apex angles the conicity must have an appreciable effect (especially with increase in time) on the temperature field near the point of application of the source. The presence of an internal cavity, even for the circular cylinder, makes a convenient analytical solution difficult, and makes necessary a solution by approximation: 1) the field in a thick-walled cylinder is calculated as for a solid cylinder [4]; 2) the field in a thin-walled cylinder is calculated as for a plane wall [4].

It is of interest to evaluate quantitatively the effect of conicity and an internal cavity (both for a cylinder and for a cone) on the temperature field obtained according to the above schemes.

Hollow cylinder: On the basis of the theorem concerning the resolution of the heat conduction equation into orthogonal components [5], the process of propagation in a hollow circular cylinder of heat from an annular concentrated source may be written as the product of the two functions $K(z, \tau)$ and $\Phi(r, \tau)$.

$K(z, \tau)$ is not affected by the presence of an internal cavity and may easily be calculated in accordance with [4]. $\Phi(r, \tau)$ has been determined with the aid of electric models or R-networks [6], since analytical evaluation of this function presents considerably difficulty.

The parameters of the R-networks were calculated so as to represent the equation of unsteady heat conduction in a cylindrical system of coordinates (case of axial symmetry).

The heat source is assumed to be uniformly distributed in a surface volume

$$V_0 = 0.2\pi[R - 0.05(R - r_1)](R - r_1).$$

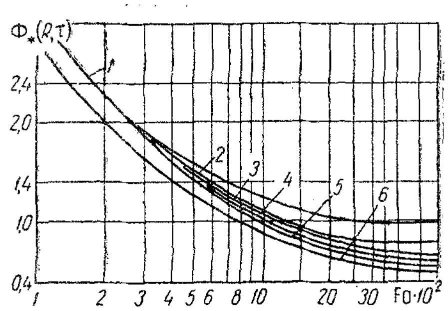

Fig. 1. $\Phi_* = \Phi(R, \tau) \times (1 - r_1)$ as a function of $\mathrm{Fo} = \frac{a\tau}{(R - r_1)^2}$: $1 - r_1/R = 0$; 2 – 0.25; 3 – 0.5; 4 – 0.6; 5 – 0.75; 6 – 1.0. Curves 1 and 6 were obtained in accordance with [4] and by electric modeling.

As in [7], the instantaneous source q" per unit length of cylinder can be modeled by an initial excess temperature of the volume, or in the case examined here by a value of function $\Phi(r, \tau)$ equal to:

$$\Phi(R, 0) = q''/c\gamma v_0.$$

The solution was found for values of r_1/R between 0 and 1. The electric modeling data for the cases $r_1/R = 0$ and $r_1/R = 1$ coincide with those calculated according to [4] (Fig. 1).

Fig. 1 shows that heat propagation along the radius for small Fo values depends only slightly on r_1/R. With increase in the ratio r_1/R, the value of the function $\Phi(R, \tau)(1 - r_1/R)$ decreases, not only due to the growth of r_1/R, but also because, for identical cylinder wall thicknesses, less heat arrives per unit area of cross section at large r_1/R. In the limit, when $\mathrm{Fo} \to \infty$, the quantity

$$\Phi(R, \tau)\left(1 - \frac{r_1}{R}\right) \to \frac{1}{1 + r_1/R},$$

and $\Phi(R, \tau) \to \frac{1}{1-(r_1/R)^2}$ under conditions of no heat transfer at the boundaries.

It is seen from Fig. 1, that, starting from Fo > 0.07 with $r_1/R > 0.6$, computation by the "plane wall" scheme will give more accurate results than computation by the "solid cylinder" scheme. Accuracy of calculation by the "plane wall" scheme may be increased if values of $\Phi(r, \tau)$ calculated according to [4] are multiplied by the ratio $(R - r_1)/\delta$, where δ is the reduced wall thickness, defined by equating the cross-sectional areas of the hollow cylinder and the corresponding plate of thickness $2\pi R$. $\delta = (R^2 - r_1^2)/2R$.

The results of such a recomputation are given in Fig. 2, from which it may be seen that the introduction of the correction affords sufficiently accurate solution by the "plane wall" scheme in the range $r_1/R > 0.6$ and Fo ≥ 0.02.

Figs. 1 and 2 show that, for small r_1/R and large $R-r_1$, it is convenient to adopt the "solid cylinder" scheme; for Fo > 0.25 the "plane wall" scheme should be used, taking into account the above correction.

The results of electric modeling have shown that, for internal cooling, when Bi $= a(R - r_1)/\lambda < 20$, the qualitative picture is maintained, and the above recommendations therefore remain in force.

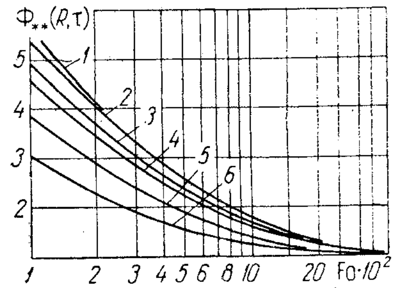

Fig. 2. $\Phi_{**} = \Phi(R, \tau) \times (1 - r_1^2/R^2)$ as a function of $\mathrm{Fo} = \frac{a\tau}{(R-r_1)^2}$: 1 – r_1/R = 1.0; 2 – 0.75; 3 – 0.6; 4 – 0.5; 5 – 0.25; 6 – 0

Cone: Let us examine the temperature field of a hollow cone with apex angles 2φ and 2Ψ (Fig. 3).

The instantaneous annular source is applied at the cross section with an outside radius R (Fig. 3). The point lies on a circle of radium R. The wall thickness at this section is

$$\delta = R\sin(\varphi - \psi)/\sin\varphi.$$

For a solid cone, $\Psi = 0$, $\delta = R$. The temperature field was obtained on an electric model or R-network, the parameters of which were calculated for a nonstationary heat conduction equation in spherical coordinates (case of spherical symmetry). There is no heat transfer at the outer and inner surfaces.

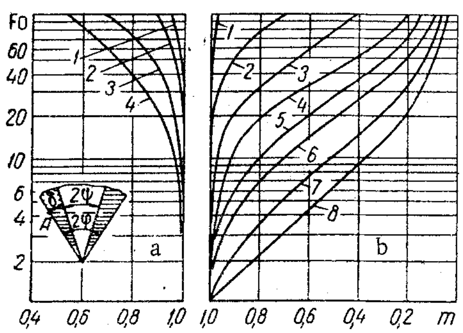

Fig. 3. Ratio m as a function of Fo for a) hollow ($\Psi = 0.75\ \varphi$; 1 – φ = 0.5; 2 – 0.6; 3 – 0.8; 4 – 1) and b) solid cone (1 – φ = 0.1; 2 – 0.2; 3 – 0.3; 4 – 0.4; 5 – 0.5; 6 – 0.6; 7 – 0.8; 8 – 1.0)

The problem is twofold. In the region where the source is applied, a large temperature drop occurs during the initial period of heating. This causes a decrease in the space and time intervals. The increase in the number of nodes of the R-network and in the steps of the solution in time requires considerable time to be spent on the solution if the usual method [6, 7] is employed. It should be noted that at the majority of nodes, measurements are made only to set up the "initial" conditions for the next step, since in analyzing the results the solutions are limited by the considerably smaller number of points in space and time. Therefore a simplified form of the semi-automatic method of solution proposed in [6] was employed (Fig. 4). To each R-network node 0 we connected a "time" circuit, consisting of a resistance R, two potentiometers (A and B), a null galvanometer, and a two-position switch. At the nodes to which "time" circuits were connected, the procedure for solution was as follows.

At the start of a solution, points 1 and 2, and 3 and 4 were connected by the switch, i.e., a voltage $V_{0,0}$, corresponding to the initial temperature, was applied to end 4 of resistor R_τ by means of potentiometer A, and, using the null galvanometer (NG), the voltage $V_{0,1}$ at point 0 was recorded at end 2 of potentiometer B. Then points 4 and 2, 3 and 1 were connected using the switch, and the process was repeated for the second moment of time, and so on.

At the nodes where "time" circuits were not connected, the voltage was measured in the usual way [8], i.e., using EGDA-type integrating circuit equipment [9]. At the nodes where "time" circuits were connected, the voltage may also be measured in the usual way in case of necessity.

Thus, the "time" circuit allows a voltage $V_{0,n}$ to be assigned to node 0 across resistor R_τ, and the voltage $V_{0,n+1}$ at node 0 to be recorded, i.e., the operation of measuring the voltage at node 0 at the n-th moment of time and the operation of assigning a voltage to R_τ at the (n+1)-th moment are combined.

Fig. 3 gives curves of the ratio m of the temperature at point A for a cone to the temperature at that point for a hollow cylinder, with outside radius R and wall thickness δ, as a function of Fo, φ and Ψ.

$$\mathrm{Fo} = a\tau/\delta^2.$$

It is seen from Fig. 3 that, for small φ and Fo, the conicity has practically no influence on the temperature field near the source, in comparison with the field in the corresponding cylinder.

When the cone has an internal cavity $\Psi \neq 0$, the error arising from disregarding conicity decreases. However, for large φ and Fo, even for thin-walled cones, the difference between the temperature fields of the cone and the cylinder is great enough to make it incorrect to disregard the conicity. The curves of Fig. 3 may be used to determine the temperature in the region of the source for cones, if the corresponding values of the temperature for the cylinder are determined from the curves of Fig. 2.

Example: To determine the temperature at point A (Fig. 3) at times τ_1 = 10 sec and τ_2 = 200 sec, if φ = 0.8, $\Psi = 0.75 \times \varphi = 0.6$, R = 0.1 m, q = 167.6 kj, λ = 41.86 w/m · degree, γ = 7,900 kj/m^3, c = 0.460 kj/kg · degree.

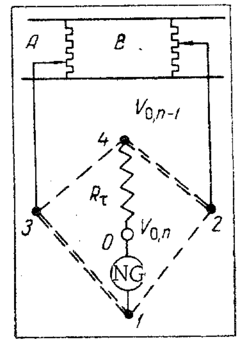

Fig. 4. Diagram of "time" circuit.

The corresponding thickness of the wall of the cone in the region of point A is

$$\delta = 0.0278 \text{ м}.$$

The values of the Fo number at times τ_1 and τ_2 are as follows:

$$\mathrm{Fo}_1 = \tau_1 \lambda / c\gamma\delta^2 = 0.1489; \ \mathrm{Fo}_2 = \tau_2 \lambda / c\gamma\delta^2 = 2.978.$$

We shall determine the temperature for corresponding dimensions of the hollow cylinder.

$$r_1 = R - \delta = 0.0722 \text{ м}; \ r_1/R = 0.722.$$

From Fig. 1 or 2, we find $\Phi(R, \tau_1) = 3.133$; $\Phi(R, \tau_2) = 2.089$.
According to [4], $K(z, \tau)$ for z = 0 has the form

$$K(0, \tau) = q'/\sqrt{\pi\lambda c\gamma\tau} = q/2\pi\frac{3}{2}R^2\sqrt{\lambda c\gamma\tau}.$$

After substitution we have $K(0, \tau) = 122/\sqrt{\tau}$ °C, i.e., $K(0,\tau_1)$ = 38.6°C; $K(0, \tau_2)$ = 8.6°C. Correspondingly, the temperature at point A for a hollow cylinder is

$$T_c(A, \tau_1) = K(0, \tau_1)\Phi(R, \tau_1) = 120.9^\circ \mathrm{C},$$
$$T_c(A, \tau_2) = K(0, \tau_2)\Phi(R, \tau_2) = 18.0^\circ \mathrm{C}.$$

From the curves of Fig. 3 we find coefficient m.

For τ_1 m = 1.0; for τ_2 m = 0.94. Thus, for the cone

$$T_k(A, \tau_1) = 120.9^\circ \mathrm{C}; \ T_k(\mathrm{A}, \tau_2) = 16.9^\circ \mathrm{C}.$$

NOTATION

$K(z, \tau)$ – a function describing the temperature distribution in an infinite cylinder with an insulated lateral surface due to a plane instantaneous heat source of intensity $q' = q/\pi R^2$; $\Phi(r,\tau)$ uniformly distributed in the section z = 0; $\Phi(r, \tau)$ – a function describing the temperature distribution in an infinite cylinder due to an instantaneous source of intensity $q'' = \pi R^2 c\gamma$ per unti length uniformly distributed over the outer cylindrical surface of radius R; z, r – instantaneous space coordinates along the cylinder axis and radius; r – radius of internal cavity; τ – time from moment of application of instantaneous source; q – intensity of annular concentrated source; $V_{0,n}$ – voltage at node 0 of electric model at the n-th time instant – analog of temperature $T_{0,n}$; n – number of time steps.

REFERENCES

1. N. N. Rykalin, Calculation of Thermal Processes in Welding [in Russian], Mashgiz, 1951.
2. N. N. Rykalin, In the collection: Thermal phenomena in metal-cutting processes, NTO, Mashprom, 1959.
3. V. I. Makhnenko, Avtomaticheskaya svarka, No. 11, 1963.
4. A. V. Lykov, Theory of Heat Conduction [in Russian], GITTL, 1952.
5. N. N. Rykalin, DAN SSSR, **125**, No. 3, 1959.
6. G. Liebmann, Trans. ASME **78**, No. 3, 1956.
7. L. A. Kozdoba and V. I. Makhnenko, Avtomaticheskaya svarka, No. 11, 1962.
8. L. A. Kozdoba, Izv. VUZ, Energetika, No. 2, 1960.
9. P. F. Fil'chakov and V. I. Panchishin, EGDA integrators [in Russian], Izd-vo AN USSR, Kiev, 1961.

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